Int. J. Solids Structures. 1978, Vol. 14, pp. 401–406. Pergamon Press. Printed in Great Britain

FREE EDGE STRESS FIELDS IN COMPOSITE LAMINATES

N. J. PAGANO
Nonmetallic Materials Division, Air Force Materials Laboratory, Wright-Patterson AFB, OH 45433, U.S.A.

(Received 3 June 1977; revised 28 October 1977; received for publication 1 December 1977)

Abstract—A new theory to predict stress fields within composite laminates is employed to solve the free edge class of boundary value problems. In this theory, the stress field determination reduces to the solution of a one-dimensional problem. The solution involves consideration of $13N$ algebraic and ordinary differential equations, when N is the number of layers in the laminate. The general solution is valid for an arbitrary number of layers, however, numerical constraints limit the value of N which can be treated by the present approach.

INTRODUCTION

An approximate theory to define the stress field within a composite laminate has recently been developed[1]. It is the purpose of the present communication to derive the solution for a significant class of stress concentration boundary value problems in composite laminates, namely, the free edge problem[2, 3], based upon this new theory.

We consider a symmetric laminate in which each layer is reinforced by a system of parallel fibers oriented at an angle θ with the x-axis (see Fig. 1 of [1]) where the origin of coordinates is located at the center of the laminate. The body is subjected to forces applied only on the ends such that a constant axial strain $\epsilon_x = \epsilon$ is imposed. Each layer is also under the influence of expansional strains e_β ($\beta = 1, 2, 3, 6$) which we assume are constants. We also assume perfect bonding between adjacent layers. Hence, the stress field in this class of problems is a function of y and z alone. The laminate consists of N layers which are identified by the index k ($k = 1, 2, \dots N$). As in [1], we shall drop the index k except when it is needed for clarity.

To begin, we define the deformation measures ϵ_i, κ_β, and expansional deformations α_β as

$$\epsilon_1 = \frac{h}{2}\bar{u}_{,x}, \quad \epsilon_2 = \frac{h}{2}\bar{v}_{,y}, \quad \epsilon_3 = 3w^*, \quad \epsilon_6 = \frac{h}{2}(\bar{u}_{,y} + \bar{v}_{,x})$$

$$\epsilon_4 = \frac{5h}{8}(\bar{w}_{,y} - \hat{w}_{,y}) + \frac{5v^*}{2}, \quad \epsilon_5 = \frac{5h}{8}(\bar{w}_{,x} - \hat{w}_{,x}) + \frac{5u^*}{2}$$

$$\kappa_1 = \frac{h^2}{4}u^*_{,x}, \quad \kappa_2 = \frac{h^2}{4}v^*_{,y}, \quad \kappa_3 = \frac{5}{4}h(3\hat{w} - \bar{w}), \quad \kappa_6 = \frac{h^2}{4}(u^*_{,y} + v^*_{,x})$$

$$\alpha_\beta = he_\beta \tag{1}$$

where lower case Latin subscripts have the range 1–6 and Greek subscripts assume the values 1–3, 6. Letting S_{ij} represent the (monoclinic) compliance matrix as given by eqn (12) of [1], we make the further definitions

$$A_{ij} = \left[S_{ij} + \frac{1}{5}\delta_{3i}\delta_{3j}S_{33}\right]^{-1} \qquad B_{ij} = \left[S_{ij} + \frac{3}{7}\delta_{3i}\delta_{3j}S_{33}\right]^{-1} \tag{2}$$

where δ_{ij} is the Kronecker delta and the superscript -1 stands for matrix inverse. Through this work, summation over the range of repeated subscripts, but not superscripts, is implied. We may now invert the constitutive relations, (17) of [1], to get

$$N_\alpha = A_{\alpha\beta}(\epsilon_\beta - \alpha_\beta) + \frac{A_{3\alpha}S_{33}h}{10}(p_1 + p_2)$$

$$M_\alpha = B_{\alpha\beta}\kappa_\beta + \frac{B_{3\alpha}S_{33}h^2}{28}(p_2 - p_1)$$

$$V_y = A_{44}\epsilon_4 + A_{45}\epsilon_5 + \frac{h}{12}(s_1 + s_2)$$

$$V_x = A_{45}\epsilon_4 + A_{55}\epsilon_5 + \frac{h}{12}(t_1 + t_2) \tag{3}$$

where standard contracted notation has been employed in the first two expressions, following the subscript relations given in (8) and (9) of [1].

Since the stresses in the present class of boundary value problems are independent of x, it follows that the force and moment resultants and the interlaminar stresses are also independent of x. Hence, from (3), the deformation measures are functions of y alone and by use of eqns (1), it can be shown that the most general form of the weighted displacements is given by

$$
\begin{aligned}
\bar{u} &= U(y) + c_1xy + c_2x \\
\bar{v} &= V(y) - \frac{c_1x^2}{2} + c_3x \\
h\bar{w} &= W(y) - 6c_5xy - 3c_6x^2 + 3c_4x \\
u^* &= \psi(y) + c_6x \\
v^* &= \Omega(y) + c_5x \\
w^* &= \phi(y) \\
h\hat{w} &= \chi(y) - 2c_5xy - c_6x^2 + c_4x
\end{aligned} \tag{4}
$$

where U---χ are arbitrary functions of y and c_1---c_6 are constants. We should recall that eqns (1)–(4) must be written for *each layer.*

We now substitute eqns (4) into (1), thence into (3), and finally into the equilibrium equations, (18) of [1], in which the x dependence is dropped to establish the following relations

$$
\begin{aligned}
&\frac{A_{22}h}{2}V'' + 3A_{23}\phi' + \frac{A_{26}h}{2}U'' + \frac{A_{23}S_{33}h}{10}(p_1' + p_2') + s_2 - s_1 = -\frac{A_{12}h}{2}c_1 \\
&\frac{A_{26}h}{2}V'' + 3A_{36}\phi' + \frac{A_{66}h}{2}U'' + \frac{A_{36}S_{33}h}{10}(p_1' + p_2') + t_2 - t_1 = -\frac{A_{16}h}{2}c_1 \\
&\frac{B_{22}h^2}{4}\Omega'' + \frac{5}{4}B_{23}(3\chi' - W') + \frac{B_{26}h^2}{4}\psi'' - \frac{5}{2}A_{45}\psi - \frac{5}{8}A_{44}(W' - \chi' + 4\Omega) \\
&\quad + \frac{B_{23}S_{33}h^2}{28}(p_2' - p_1') + \frac{5h}{12}(s_1 + s_2) = \frac{5}{4}A_{45}(c_4 - 2c_5y) \\
&\frac{B_{26}h^2}{4}\Omega'' + \frac{5B_{36}}{4}(3\chi' - W') + \frac{B_{66}h^2}{4}\psi'' - \frac{5A_{55}}{2}\psi - \frac{5}{8}A_{45}(W' - \chi' + 4\Omega) \\
&\quad + \frac{B_{36}S_{33}h^2}{28}(p_2' - p_1') + \frac{5h}{12}(t_1 + t_2) = \frac{5}{4}A_{55}(c_4 - 2c_5y) \\
&\frac{A_{23}h}{2}V' + 3A_{33}\phi + \frac{A_{36}h}{2}U' + \frac{h}{10}(A_{33}S_{33} - 5)(p_1 + p_2) + \frac{h^2}{12}(s_1' - s_2') \\
&\quad = -\frac{A_{13}h}{2}(c_1y + c_2) - \frac{A_{36}h}{2}c_3 + A_{3\beta}\alpha_\beta \\
&\frac{5A_{45}}{2}\psi' + \frac{5}{8}A_{44}(W'' - \chi'' + 4\Omega') + p_2 - p_1 + \frac{h}{12}(s_1' + s_2') = \frac{5}{2}A_{45}c_5 \\
&\frac{5B_{23}}{2}\Omega' + \frac{25B_{33}}{2h^2}(3\chi - W) + \frac{5B_{36}}{2}\psi' + \frac{1}{14}(5B_{33}S_{33} - 14)(p_2 - p_1) - \frac{h}{12}(s_1' + s_2') \\
&\quad = -\frac{5B_{13}c_6}{2} - \frac{5B_{36}c_5}{2}
\end{aligned} \tag{5}
$$

which are also valid within each layer. The remaining field equations, the interface continuity conditions, are given by substituting (4) into (19) of [1], which gives

$$
\left\{V - \frac{h}{2}\phi' - \frac{5}{2}\Omega + \frac{5}{8}\chi' - \frac{W'}{8} - \frac{h}{12}[S_{44}(3s_1 - s_2) + S_{45}(3t_1 - t_2)]\right\}^{(k+1)}
$$

$$-\left\{V-\frac{h}{2}\phi'+\frac{5}{2}\Omega-\frac{5}{8}\chi'+\frac{W'}{8}+\frac{h}{12}[S_{44}(3s_2-s_1)+S_{45}(3t_2-t_1)]\right\}^{(k)}=3(\overset{(k+1)}{c_5}+\overset{(k)}{c_5})x$$
$$+(\overset{(k)}{c_3}-\overset{(k+1)}{c_3})x+\frac{1}{2}(\overset{(k+1)}{c_1}-\overset{(k)}{c_1})x^2$$

$$\left\{U-\frac{5}{2}\psi-\frac{h}{12}\left[S_{45}(3s_1-s_2)+S_{55}(3t_1-t_2)\right]\right\}^{(k+1)}-\left\{U+\frac{5}{2}\psi+\frac{h}{12}[S_{45}(3s_2-s_1)\right.$$
$$\left.+S_{55}(3t_2-t_1)]\right\}^{(k)}=\frac{1}{2}(\overset{(k+1)}{c_5}+\overset{(k)}{c_5})y+(\overset{(k)}{c_1}-\overset{(k+1)}{c_1})xy+3(\overset{(k+1)}{c_6}+\overset{(k)}{c_6})x+(\overset{(k)}{c_2}-\overset{(k+1)}{c_2})x-\frac{1}{4}(\overset{(k+1)}{c_4}+\overset{(k)}{c_4})$$

$$\overset{(k+1)}{S_{33}}\left\{-\frac{7}{2}A_{23}hV'+21\left(\frac{5}{S_{33}}-A_{33}\right)\phi-\frac{7}{2}A_{36}hU'+\frac{15}{2}B_{23}h\Omega'+\frac{75}{2h}\left(3B_{33}-\frac{7}{S_{33}}\right)\chi\right.$$
$$+\frac{15}{2h}\left(\frac{7}{S_{33}}-5B_{33}\right)W+\frac{15}{2}B_{36}h\psi'+hp_1\left(6-\frac{7A_{33}S_{33}}{10}-\frac{15B_{33}S_{33}}{14}\right)$$
$$\left.+hp_2\left(1-\frac{7A_{33}S_{33}}{10}+\frac{15B_{33}S_{33}}{14}\right)\right\}^{(k+1)}+\overset{(k)}{S_{33}}\left\{-\frac{7}{2}A_{23}hV'+21\left(\frac{5}{S_{33}}-A_{33}\right)\phi-\frac{7}{2}A_{36}hU'-\frac{15}{2}B_{23}h\Omega'\right.$$
$$+\frac{75}{2h}\left(\frac{7}{S_{33}}-3B_{33}\right)\chi+\frac{15}{2h}\left(5B_{33}-\frac{7}{S_{33}}\right)W-\frac{15}{2}B_{36}h\psi'+hp_1\left(1-\frac{7A_{33}S_{33}}{10}\right.$$
$$\left.\left.+\frac{15B_{33}S_{33}}{14}\right)+hp_2\times\left(6-\frac{7A_{33}S_{33}}{10}-\frac{15B_{33}S_{33}}{14}\right)\right\}^{(k)}=105\left[2\left(\frac{\overset{(k)}{c_5}}{h_k}-\frac{\overset{(k+1)}{c_5}}{h_{k+1}}\right)xy\right.$$
$$\left.+\left(\frac{\overset{(k+1)}{c_4}}{h_{k+1}}-\frac{\overset{(k)}{c_4}}{h_k}\right)x+\left(\frac{\overset{(k)}{c_6}}{h_k}-\frac{\overset{(k+1)}{c_6}}{h_{k+1}}\right)x^2\right]+\frac{7y}{2}(\overset{(k+1)}{S_{33}}\overset{(k+1)}{A_{13}}h_{k+1}\overset{(k+1)}{c_1}$$
$$+\overset{(k)}{S_{33}}\overset{(k)}{A_{13}}h_k\overset{(k)}{c_1})+\overset{(k+1)}{S_{33}}\left(\frac{7}{2}A_{13}hc_2+\frac{7}{2}A_{36}hc_3-\frac{15}{2}B_{13}hc_6\right.$$
$$\left.-\frac{15}{2}B_{36}hc_5-7A_{3\beta}\alpha_\beta\right)^{(k+1)}+\overset{(k)}{S_{33}}\left(\frac{7}{2}A_{13}hc_2+\frac{7}{2}A_{36}hc_3+\frac{15}{2}B_{13}hc_6+\frac{15}{2}B_{36}hc_5-7A_{3\beta}\alpha_\beta\right)^{(k)}\tag{6}$$

and

$$\overset{(k+1)}{t_1}=\overset{(k)}{t_2},\quad \overset{(k+1)}{s_1}=\overset{(k)}{s_2},\quad \overset{(k+1)}{p_1}=\overset{(k)}{p_2}\tag{7}$$

for $k=1,2\ldots N-1$.

Owing to the symmetric lamination geometry, the interlaminar shear stress components and the z displacement component, w, all vanish on the central plane $z=0$. We shall take advantage of these conditions by considering only the upper half of the laminate, i.e. $z\geq 0$. Incorporating the traction-free conditions on the upper surface, our boundary conditions on the upper and lower surfaces become

$$\overset{(1)}{S_{33}}\left\{-\frac{7}{2}A_{23}hV'+21\left(\frac{5}{S_{33}}-A_{33}\right)\phi-\frac{7}{2}A_{36}hU'+\frac{15}{2}B_{23}h\Omega'+\frac{75}{2h}\left(3B_{33}-\frac{7}{S_{33}}\right)\chi\right.$$
$$+\frac{15}{2h}\left(\frac{7}{S_{33}}-5B_{33}\right)W+\frac{15}{2}B_{36}h\psi'+hp_1\left(6-\frac{7A_{33}S_{33}}{10}-\frac{15B_{33}S_{33}}{14}\right)$$
$$\left.+hp_2\left(1-\frac{7A_{33}S_{33}}{10}+\frac{15B_{33}S_{33}}{14}\right)\right\}^{(1)}=\frac{105}{h_1}(-2\overset{(1)}{c_5}xy+\overset{(1)}{c_4}x-\overset{(1)}{c_6}x^2)$$
$$+\overset{(1)}{S_{33}}\left(\frac{7}{2}A_{13}hc_1y+\frac{7}{2}A_{13}hc_2+\frac{7}{2}A_{36}hc_3-\frac{15}{2}B_{13}hc_6-\frac{15}{2}B_{36}hc_5-7A_{3\beta}\alpha_\beta\right)^{(1)}\tag{8}$$

and

$$\overset{(1)}{t_1}=\overset{(1)}{s_1}=\overset{(N)}{t_2}=\overset{(N)}{s_2}=\overset{(N)}{p_2}=0.\tag{9}$$

Since eqns (6) and (8) must be satisfied for all values of x, it follows that

$$\overset{(k)}{c}_4 = \overset{(k)}{c}_5 = \overset{(k)}{c}_6 = 0, \quad k = 1, 2 \dots N \tag{10}$$

and

$$\overset{(k+1)}{c}_1 = \overset{(k)}{c}_1, \quad \overset{(k+1)}{c}_2 = \overset{(k)}{c}_2, \quad \overset{(k+1)}{c}_3 = \overset{(k)}{c}_3, \quad k = 1, 2 \dots N - 1 \tag{11}$$

so that c_1, c_2 and c_3 are the same for every layer.

We now turn our attention to the edge boundary conditions, which require consideration of N_y, N_{xy}, V_y, M_y, M_{xy}, s_1 and s_2 for each layer on $y = \pm b$, since, as discussed in [1], only edge tractions are imposed in the present class of boundary value problems. However, all these functions cannot be independently prescribed because of the consequences of interface continuity and global equilibrium of the entire laminate. That is, the interface continuity conditions given by the second of (7) prohibit arbitrarily prescribed values of $\overset{(k)}{s}_1$ and $\overset{(k)}{s}_2$. Furthermore, $\overset{(1)}{s}_1$ and $\overset{(N)}{s}_2$ have already been specified by (9) for all values of y. These relations, in conjunction with the second equilibrium equation, see (18) of [1], can be used to establish the result

$$\sum_{k=1}^{N} \overset{(k)}{N}_{y,y} = 0 \tag{12}$$

which requires that

$$\sum_{k=1}^{N} \overset{(k)}{N}_y(b) - \sum_{k=1}^{N} \overset{(k)}{N}_y(-b) = 0. \tag{13}$$

Therefore, only $2N - 1$ values of $\overset{(k)}{N}_y$ can be arbitrarily prescribed on the edges $y = \pm b$. We can make the same statement regarding $\overset{(k)}{N}_{xy}$ since an equation of the form (13) can be derived in similar fashion for this function. Hence, the edge boundary conditions may be expressed as

$$\begin{gathered} \overset{(k)}{N}_y(b) = \overset{(k)}{N}_{xy}(b) = \overset{(k)}{V}_y(\pm b) = \overset{(k)}{M}_y(\pm b) = \overset{(k)}{M}_{xy}(\pm b) = 0 \quad (k = 1, 2 \dots N) \\ \overset{(k)}{N}_y(-b) = \overset{(k)}{N}_{xy}(-b) = \overset{(k)}{s}_2(\pm b) = 0 \quad (k = 1, 2 \dots N - 1). \end{gathered} \tag{14}$$

The present boundary value problem therefore, consists of differential equations (5) and (6) subject to the boundary conditions (8), (9) and (14).

The general solution for each dependent variable consists of the sum of two parts: (i) a complementary solution defined by the homogeneous form of (5)–(9) and (ii) a particular solution. In the particular solution (denoted by subscript P), the only non-vanishing functions are given by

$$\overset{(k)}{\phi}_P = \overset{(k)}{a}_1 \qquad \overset{(k)}{\chi}_P = \overset{(k)}{a}_2 \qquad \overset{(k)}{W}_P = 3\overset{(k)}{a}_2 \tag{15}$$

where $\overset{(k)}{a}_i$ $(i = 1, 2)$ are constants given by substituting (15) into (5), (6) and (8) to get

$$\begin{aligned} \overset{(k)}{a}_1 &= \frac{1}{3A_{33}^{(k)}}(\overset{(k)}{A}_{3\beta}\overset{(k)}{\alpha}_\beta - \overset{(k)}{A}_{13}h_k\epsilon), \quad k = 1, 2 \dots N \\ \overset{(1)}{a}_2 &= h_1\overset{(1)}{a}_1 \end{aligned} \tag{16}$$

$$\frac{\overset{(k+1)}{a}_2}{h_{k+1}} = \overset{(k+1)}{a}_1 + \overset{(k)}{a}_1 + \frac{\overset{(k)}{a}_2}{h_k}, \qquad k = 1, 2 \dots N-1$$

where we have put

$$2c_2 = \epsilon, \quad c_1 = c_3 = 0 \tag{17}$$

and ϵ is the applied axial strain. The second condition, $c_1 = 0$, follows from the procedure leading to (15) and (16), which c_3 is set to zero since it has no effect on the stress field here.

Since the field equations are linear differential equations with constant coefficients, the complementary solution (subscript H) for each dependent variable consists of a series of terms of the form

$$\overset{(k)}{f}_H = \overset{(k)}{F} \mathrm{e}^{\lambda y} \tag{18}$$

where $\overset{(k)}{f}$ stands for any of the dependent variables and $\overset{(k)}{F}$ are constants. Substituting (18) into the homogeneous form of eqns (5)–(9) leads to a system of $13N$ linear algebraic equations. The values of λ are determined by setting the determinant of the coefficients equal to zero. Algebraic expressions for the expansion of the determinant were not written owing to the complexity of such expressions, even in the simplest cases. Rather, computer calculated values of the determinant for specific values of λ, as discussed later, were employed to continue the analysis. Hence, it is not possible to exhibit the mathematical details of the remaining steps in the solution. This phase of the work, therefore, will be presented in descriptive fashion.

Although our determinant is too complex to evaluate in general terms, we can examine the nature of its polynomial (in λ) expansion in some detail for fairly small values of N and extend the results by induction.† We can also develop a procedure to search for terms in the determinant in which the highest and lowest powers of λ occur. Proceeding in this fashion for values of $N = 1$ (in which case the interface continuity conditions are dropped) 2, 3 and 4, we make the following observations: (i) Only even powers of λ occur in the determinant, (ii) The lowest power of λ is λ^4, and (iii) The highest power of λ is λ^{12N-2}. Although these results are not perfectly rigorous, no contradictions have been found. Further, the number of λ roots is consistent with the number of edge boundary conditions, as discussed later.

An exception to the form (18) occurs with the appearance of repeated roots for λ. Since repeated zero roots always occur, it is necessary to treat the corresponding part of the solution separately. In this part of the solution, (18) is replaced by a third degree polynomial in y. Representing the functions corresponding to the repeated zero roots by subscripts Ho, we find from the homogeneous form of (5)–(9) that the only non-vanishing functions are given by

$$\overset{(k)}{U}_{\mathrm{Ho}} = C_1 y + C_0 \qquad \overset{(k)}{V}_{\mathrm{Ho}} = A_1 y + A_0 \qquad \overset{(k)}{\phi}_{\mathrm{Ho}} = \overset{(k)}{B}_0 \qquad \overset{(k)}{\chi}_{\mathrm{Ho}} = \overset{(k)}{E}_0 \qquad \overset{(k)}{W}_{\mathrm{Ho}} = 3\overset{(k)}{E}_0 \tag{19}$$

where

$$\overset{(k)}{B}_0 = -\frac{(\overset{(k)}{A}_{23} A_1 + \overset{(k)}{A}_{36} c_1) h_k}{6A_{33}^{(k)}}, \quad k = 1, 2 \dots N \qquad \overset{(1)}{E}_0 = h_1 \overset{(1)}{B}_0$$

$$\frac{\overset{(k+1)}{E}_0}{h_{k+1}} = \overset{(k)}{B}_0 + \frac{\overset{(k)}{E}_0}{h_k} + \overset{(k+1)}{B}_0, \qquad k = 1, 2 \dots N-1 \tag{20}$$

The constants A_0 and C_0 define rigid body translation of the laminate as a unit. The remaining

†Although we have not done it here, it is possible to develop a program to define the powers of λ occurring in each term of the determinant for arbitrary values of N by use of a computer.

constants in (19) can all be expressed in terms of A_1 and C_1. Hence, two constants which effect the stress distribution have been introduced in the repeated zero part of the homogeneous solution.

The remaining portion of the complementary solution consists of functions of the form (18) corresponding to the $12N-6$ non-zero values of λ (we are assuming that there roots are all distinct). These roots, which occur in combinations of the form $\pm(a \pm ib)$, where b may vanish, were determined by computing the value of the determinant for $6N-2$ values of λ^2 and using these results to solve for the coefficients of the equivalent polynomial. Once the polynomial has been established, its roots can be determined through standard computer routines. In the usual manner, one equation in the homogeneous version of (5)–(9) may be dropped and the reduced system used to relate all but one of the arbitrary constants to the remaining constant for each value of λ. This procedure leads to $12N-6$ arbitrary constants. These constants, augmented by A_1 and C_1, are evaluated via the $12N-4$ edge boundary conditions (14). Once these constants are defined, the complete solution for any of the $13N$ functions appearing in (5)–(9) is given by

$$\overset{(k)}{f} = \sum_{m=1}^{12N-6} \overset{(k)}{F}_m \, e^{\lambda_m} + \overset{(k)}{f}_{\mathrm{Ho}} + \overset{(k)}{f}_P \tag{21}$$

where the last two terms are defined by (15), (16), (19) and (20). The force and moment resultants are now given by (3) upon using the results of (4) and (1).

A special case arises in the solution that requires separate treatment. This case is defined by the vanishing of compliance coefficients S_{16}, S_{26}, S_{36} and S_{45}, as well as expansional strain e_6, in *every* layer. The situation arises when each layer is isotropic and/or oriented at an angle of 0° or 90°. This leads to the vanishing of $\overset{(k)}{U}$, $\overset{(k)}{\phi}$, $\overset{(k)}{t}_1$, $\overset{(k)}{t}_2$, $\overset{(k)}{N}_{xy}$ and $\overset{(k)}{M}_{xy}$. Consequently, dropping the appropriate field equations and boundary conditions we find that setting the determinant to zero leads to only two zero roots for λ and $8N$-4 non-zero roots. The number of boundary conditions (14) reduces to $8N-3$. Otherwise, the treatment presented here remains unchanged.

The occurrence of very large magnitudes of λ for large N in the present formulation leads to values of $e^{\pm\lambda b}$ which exceed computer limits. This in turn restricts the values of N which can be treated. For example, $N=6$ was the largest value permissible for the properties assumed in [1]. Resolution of this difficulty is now being considered. Numerical results of the present solution are presented in [1].

REFERENCES

1. N. J. Pagano, Stress fields in composite laminates, *Int. J. Solids Structures* **14**, 385–400 (1978).
2. A. H. Puppo and H. A. Evensen, Interlaminar shear in laminated composites. *J. Composite Mat.* **4**, 204 (1970).
3. R. B. Pipes and N. J. Pagano, Interlaminar stresses in composite laminates under uniform axial extension. *J. Composite Mat.* **4**, 538 (1970).